Application Report
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A Numerical Solution to an Analog Problem

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ABSTRACT

In order to derive a solution for an analog circuit problem, it is often useful to develop a model. This approach is generally accepted as developing an analytical model. However, finding the analytical solution is not always practical or possible as a result of higher-degree polynomials that require further resolution, or because of the time needed to develop the solution completely. In these situations, a discrete finite approach may be necessary. This application note develops a methodology to approximate a function with an RC network. This methodology can be easily expanded to match any function and generate a discrete solution.

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1 Introduction

When faced with the problem of developing an analog circuit that matches a frequency response over frequency, we often turn to analytical analysis and simplification. The limit of this approach lies in the maximum order of the system that can easily be analyzed: a second- or third-order equation. When looking at an equalization circuit in particular, or when developing a circuit that matches a desired frequency response, a numerical solution may prove to be less time-consuming to develop than an analytic solution. In this application note, we start by developing the methodology using a first-order system as an example; we then expand it to a fourth-order system for a single-ended equalization circuit, and compare approximate accuracy to a given curve between solutions of different orders.

2 Problem Description

Measuring 100ft of Belden 8723 multi-conductor shielded twisted pair cable gave the attenuation versus frequency performance shown in Figure 1. This configuration includes the 52.4Ω matching resistors required for proper termination.

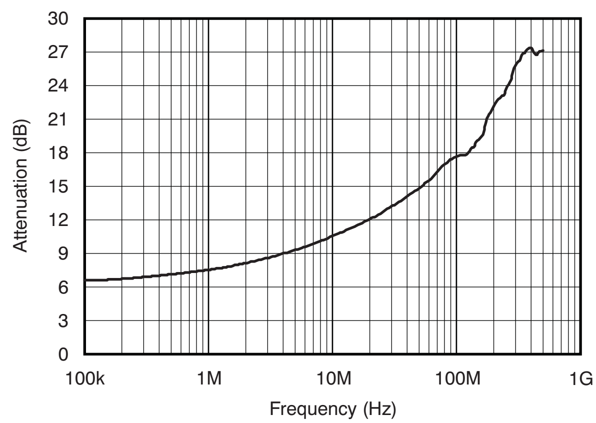

Figure 1. Belden 8723 Shielded Cable Attenuation vs Frequency Performance

Figure 2 shows a first order, single-ended equalizer circuit, in which the peaking function or equalization is accomplished by reducing the overall gain impedance.

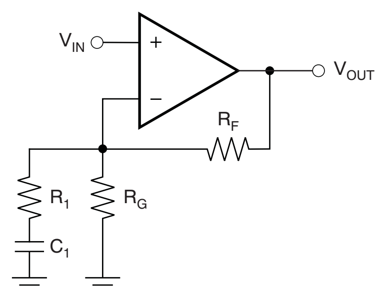

Figure 2. First-Order, Single-Ended Equalizer Circuit

This approach offers several key advantages:

- The freedom to use any operation amplifier, independent of its internal architecture.
- The pole location can be accurately set.

The gain equation is simply derived by noticing that:

$$\frac{V_{OUT}}{V_{IN}} = 1 + \frac{R_F}{Z_G}$$

$$Z_G = R_G \,||\, \left(R_1 + \frac{1}{sC_1}\right) \quad (1)$$

Simplifying this formula produces Equation 2:

$$\frac{V_{OUT}}{V_{IN}} = 1 + \frac{R_F}{Z_G} \bullet \frac{1 + s(R_1 + R_G)C_1}{(1 + sR_1C_1)} \quad (2)$$

Note that Equation 2 contains one pole and one zero.

Figure 3 shows a typical frequency response.

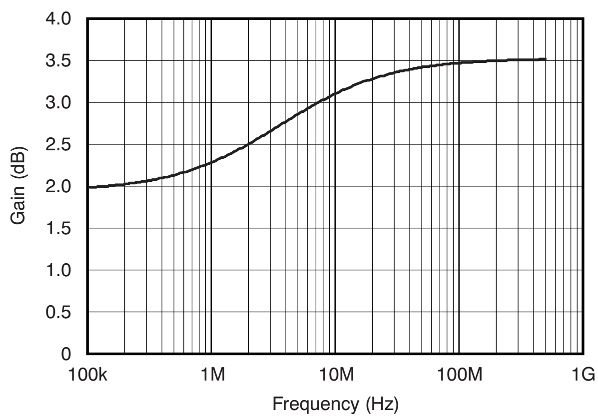

Figure 3. First-Order, Single-Ended Frequency Response Example

How then do we find a solution that approximates the curve in Figure 1 by setting R_1 and C_1 to the correct values?

3 Initial Approach: Analytical Solution

One approach (not developed here) is to notice that the attenuation can be approximated as a power function and develop a corresponding analytical solution. Figure 4 illustrates an approximation of the cable attenuation generated using Microsoft® Excel®.

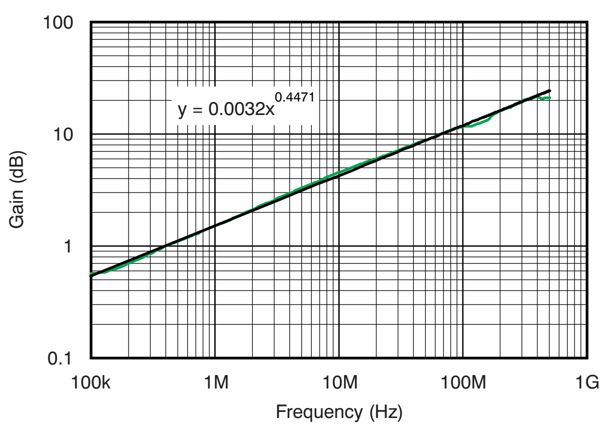

Figure 4. Power Function Equivalent to Signal Attenuation in Belden 8723 Cable

More simply, consider that each frequency point is a vector. If you have 10 frequencies in your frequency response curve, you have a 10-dimension vector.

4 A Numerical Approach

The numerical approach developed here is described below. For each frequency, calculate the formula developed in Equation 2. Subtract the result from your measured data point; then square this result so that each frequency can be added as vector. Sum all the vectors together, take the square root, and find a minimum value for this vector by changing R_G, R_1, and C_1.

Although this approach may seem complicated at first glance, we will go through it step-by-step, using a spreadsheet to do the math for us.

4.1 Creating the Spreadsheet

The first step here is to create a spreadsheet that will handle all the calculations. Table 1 shows the calculation spreadsheet created for this example.

Table 1. Calculation Spreadsheet

R_F (ohm)	**200**					
R_G	**3970.21039**					
R_1 (ohm)	**312.420357**					
C_1 (pF)	**142.401448**					
(R_1, C_1) pole location	3.58	MHz				
Frequency (Hz)	Ra+1/sCa	Zg	Gain (V/V)	Cable Attenuation (V/V)	Fit?	Relative error ^2
100000	11488.9179	2950.58171	1.067783	1.067379	1	1.63E-07
104350	11023.0073	2918.897	1.068519	1.067436	1	1.17E-06
108890	10576.446	2886.6232	1.069285	1.067747	1	2.37E-06
113627	10148.5487	2853.78291	1.070082	1.068317	1	3.12E-06
459175690	314.854393	291.719786	1.685589	11.10222	0	0
479152832	314.752911	291.632668	1.685794	11.20753	0	0
500000000	314.655657	291.549174	1.685991	11.32531	0	0
						0.036472

To minimize error, the implementation was made using Equation 1 and separating $\left(R_1 + \frac{1}{sC_1}\right)$ from the Z_G. These formulas are given in Table 2.

Table 2. Formulas Used in the Calculation Spreadsheet

R_F (ohm)	**200**					
R_G	**3970.21039**					
R_1 (ohm)	**312.420357**					
C_1 (pF)	**142.401448**					
(R_1, C_1) pole location	3.58	MHz				
Frequency (Hz)	Ra+1/sCa	Zg	Gain (V/V)	Cable Attenuation (V/V)	Fit?	Relative error ^2
f_0	$Z_0 = R_1 + (2\pi \bullet f_0 \bullet C_1 \bullet 10^{-12})^{-1}$	$Z_{g0} = \frac{Z_0 \bullet R_g}{Z_0 + R_g}$	$Gain_0 = 1 + \frac{R_f}{Z_{g0}}$	$Cable_{att\_0}$	Fit_0	$Fit_0 \bullet (Cable_{att\_0} - Gain_0)$
f_n	$Z_n = R_1 + (2\pi \bullet f_n \bullet C_1 \bullet 10^{-12})^{-1}$	$Z_g = \frac{Z_n \bullet R_g}{Z_n + R_g}$	$Gain_n = 1 + \frac{R_f}{Z_{gn}}$	$Cable_{att\_n}$	Fit_n	$Fit_n \bullet (Cable_{att\_n} - Gain_n)$
						$\sum_{t=0}^{n} Fit_t \bullet (Cable_{att\_t} - Gain_t)$

The last step of this procedure is to minimize the sum of all relative errors by adjusting a set of parameters. The parameters for this problem are:

- The gain resistor (R_G)
- The equalizing resistor and capacitor (R_1 and C_1, respectively)

Note that the feedback resistor R_F should not be considered as one of these parameters as setting it manually offers some advantages. For voltage feedback amplifiers, R_F may be used to limit the loading of the amplifier as the loading for a given load is set by (R_F || R_L). In the case of current feedback amplifiers, R_F can be set for optimal stability or bandwidth limitation.

For the following explanation, we must set the following:

$$\text{Sum} = \sum_{t=0}^{n} \text{Fit}_t \bullet (\text{Cable}_{att\_t} - \text{Gain})$$

$$\$B\$3 = R_G$$
$$\$B\$4 = R_A = R_1$$
$$\$B\$5 = C_A = C_1 \quad (3)$$

$B$3 to **$B$5** are used in the spreadsheet, as we can see in Figure 5.

	A	B	C	D	E
1					
2	Rf (ohm)	**200**			
3	Rg	**3970.21039**			
4	Ra (ohm)	**312.420357**			
5	Ca (pF)	**142.401448**			
6		3.58	MHz		
7					
8				Gain (V/V)	Gain (dB))
9	Frequency (Hz)	Ra+1/sCa	Single pole	1-pole	1-pole
10	100000	11488.9179	2950.58171	1.067783	0.569662

Figure 5. Spreadsheet Nomenclature and Example

With this set of parameters and by using the built-in solver function in Excel, it is possible to find a solution. This approach now leaves two remaining issues to explain:

1. Setting the initial conditions. This step is very important because it will allow you to find either a local minima or the overall minima for the error. The overall minima will likely tend to have better matching over a wider frequency range, and will therefore also reach a higher frequency.
2. Setting and using the Excel solver function.

4.2 Setting Initial Conditions

Consider the nature of the amplifier that we are going to use to set the feedback resistor. If the amplifier is a current-feedback amplifier, the feedback resistor is the compensation element. The feedback resistor must then be selected for optimum stability. See the individual amplifier product data sheet for recommendations.

If the amplifier used is a voltage-feedback amplifier, we have more choices in the feedback resistor value, but we may want to select it to minimize noise or avoid amplifier loading. If a high-speed operational amplifier is used, we must be careful with the parasitic elements introduced by the layout and components as well. A 10kΩ feedback resistance and a parasitic 0.2pF (standard value for parasitic capacitance as a result of the component and the layout of a 1206-size resistor) creates an 80MHz pole that may interact with the equalization circuit if matching to high frequency is desired. These considerations, taken together, define the value of the feedback resistor.

For an initial gain resistor value, the insertion loss of the cable at dc provides some direction as well as the dc gain that is intended for that circuit.

For a single pole, set R_A at several hundred ohms and C_A in the range of hundreds of pF, initially. As the number of poles increase, set the second pole resistor 5x to 10x smaller and the capacitor 2x to 5x smaller than the first poles, and so on.

4.3 Setting and Using the Excel Solver Function

The Excel solver function can be found in the *Tool-> Solver...* menu, as Figure 6 shows.

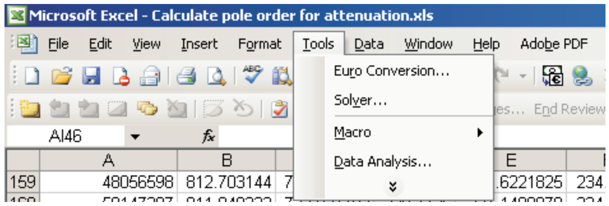

Figure 6. Finding the Solver Function

The following window (shown in Figure 7) then opens.

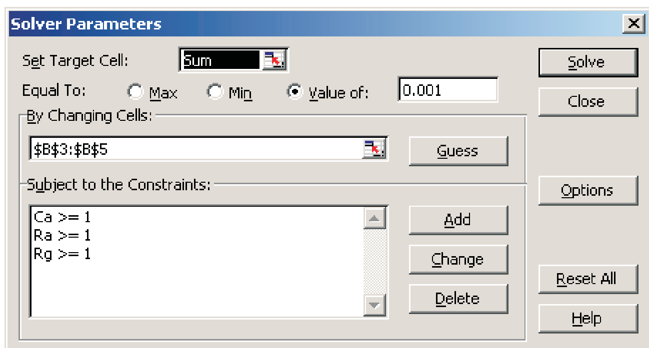

Figure 7. Solver Interface

You then select the cells that you want to optimize and the optimization method. Here, the optimization to a known value was selected. Some additional constraints to maintain the gain and equalization element positive were added to ensure that only positive values for the gain and equalization elements are found. The last element is to select what cells must be modified. ***$B$3:$B$5*** is the range selected for our example.

The solver then modifies R_G, R_A, and C_A until the sum of all the errors is as small and as close as possible to the specified target.

Figure 8 shows a single-pole solution approximation.

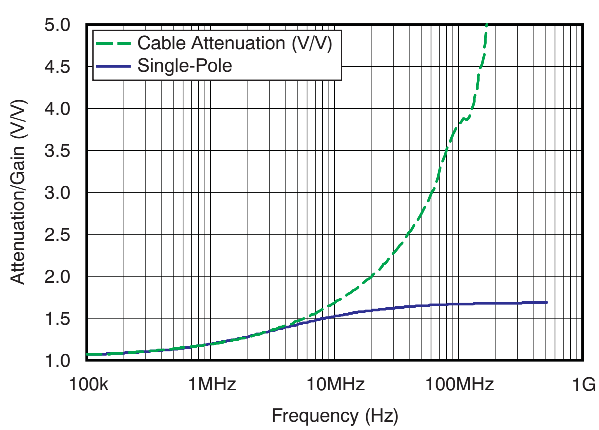

Figure 8. Single-Pole Equalization of Belden 8723 Cable

From the resulting graph, with the ideal values shown in Figure 5, we can see that a single pole will equalize the twisted-pair cable up to 3.5MHz. How then do we increase the matching to higher frequency?

5 Multiple Poles, Single-Ended Solution

Extending this approach to a multiple-pole circuit is as simple as knowing the proper equation to enter in the spreadsheet. Two-pole and three-pole equations are shown in Section 5.1 and Section 5.2, respectively. The schematic of a two-pole approach is then shown in Figure 11.

5.1 Two-Pole Solution

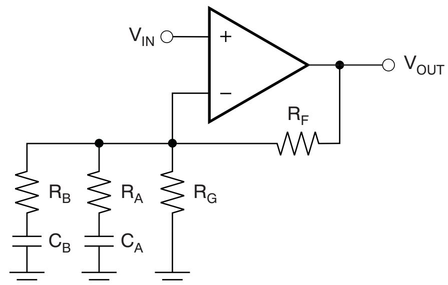

Figure 9. Second-Order, Single-Ended Equalization Circuit

Figure 9 shows a second order, single-ended equalizer circuit, in which the peaking function (or equalization) is accomplished by reducing the overall gain impedance. As with the first-order circuit (refer to Figure 2), this approach exhibits several advantages:

- The freedom to use any operation amplifier, independent of its internal architecture.
- The pole locations can be accurately set.

The gain equation is simply derived by noticing that:

$$\frac{V_{OUT}}{V_{IN}} = 1 + \frac{R_F}{Z_G}$$

$$Z_G = R_G \,||\, \left(R_A + \frac{1}{sC_A}\right) \,||\, \left(R_B + \frac{1}{sC_B}\right) \tag{4}$$

Simplifying this results in Equation 5.

$$\frac{V_{OUT}}{V_{IN}} = 1 + \frac{R_F}{Z_G} \bullet \frac{\left[1 + s(R_A + R_G)C_A\right]\bullet(1 + sR_BC_B) + sR_GC_B\bullet(1 + sR_AC_A)}{(1 + sR_AC_A)\bullet(1 + sR_BC_B)} \tag{5}$$

5.2 Three-Pole Solution

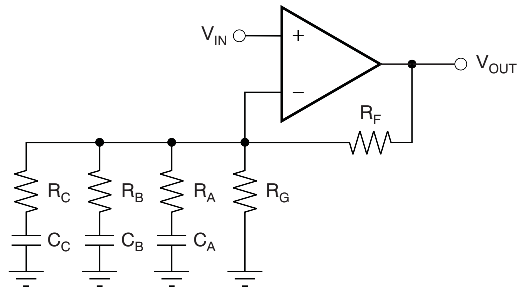

Figure 10. Third-Order, Single-Ended Equalization Circuit

Figure 10 shows a third-order, single-ended equalizer circuit, in which the peaking function or equalization is achieved by reducing the overall gain impedance. As with the first- and second-order circuits, this approach shows several advantages:

- The freedom to use any operational amplifier, independent of its internal architecture.
- The pole locations can be accurately set.

The gain equation is simply derived by noticing that:

$$\frac{V_{OUT}}{V_{IN}} = 1 + \frac{R_F}{Z_G}$$

$$Z_G = R_G \,||\, \left(R_A + \frac{1}{sC_A}\right) \,||\, \left(R_B + \frac{1}{sC_B}\right) \,||\, \left(R_C + \frac{1}{sC_C}\right) \quad (6)$$

This process is iterative from the second-order, single-ended equalization circuit shown in Equation 4, and so is the programming, as well.

6 Example Result with Three-Pole, Single-Ended Circuit (Belden 8723 Cable)

A third-order pole is sufficient to achieve matching to 100MHz. Calculation results are shown in Figure 11.

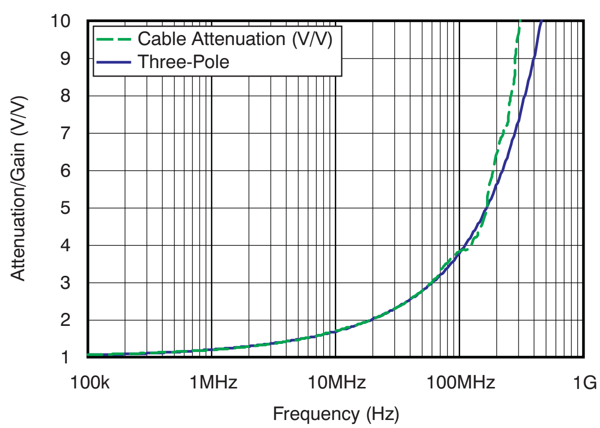

Figure 11. Third-Order, Single-Ended Equalization Circuit

The placement for the pole is obtained from the spreadsheet developed earlier. Results are shown in Figure 12.

	A	B	C	D	E
1					
2	Rf (ohm)	**200**			
3	Rg	**6608.15524**		1.243805	
4	Ra (ohm)	**792.19938**		**50.00**	MHz
5	Ca (pF)	**161.522676**			
6	Rb (ohm)	**219.819398**		27.36	MHz
7	Cb (pF)	**26.4582451**			
8	Rc (ohm)	**1**		11221.98	MHz
9	Cc (pF)	**14.182429**			
10					

Figure 12. Modeled Pole Location

7 Conclusion

This application note develops a simple numerical approach to finding a solution to complex analytical modeling problems. Emphasis on modeling frequency response and equalization is provided.